Time Allowed : 3 Hours **MATHEMATICS** **Max. Marks :80**

Instructions:

1. **All the questions are compulsory.**
2. **The question paper consists of 16 questions divided into 4 sections A,B,C and D.**
3. **Section A comprises of 3 questions :**
   - **(i)** **Q.No.1 consists of 16 Multiple Choice Questions carrying 1 mark each.**
   - **(ii)** **Q.No.2 consists of 8 Fill in the Blank type questions with options carrying 1 mark each.**
   - **(iii)** **Q.No.3 consists of 8 True/False type questions carrying 1 mark each.**
4. **Section B comprises of 5 questions of 2 marks each.**
5. **Section C comprises of 5 questions of 4 marks each.**
6. **Section D comprises of 3 questions of 6 marks each.**
7. **There is no overall choice. However, an internal choice has been provided in three questions of 2 marks, three questions of 4 marks and three questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.**
8. **Use of calculator is not permitted.**

Section – A

Q1 **Choose the correct options in the following questions :**

(i) Function $f: R \to R,\ f(x) = 3x - 5$ is : 1

(a)one-one only (b)onto only (c)one-one and onto (d)none of these

(ii) Relation given by $R = \{(1,1),(2,2),(1,2),(2,1)\}$ is 1

(a)reflexive only (b)symmetric only (c)transitive only (d) equivalence relation

(iii) $\cos^{-1}\left(-\cos\frac{2\pi}{3}\right)$ is equal to : 1

(a)$\frac{\pi}{5}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

(iv) If $\begin{bmatrix} 1 & -x \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 4 & -3 \end{bmatrix}$ then value of x is: 1

(a)8 (b)-4 (c)3 (d)-8

(v) If order of matrix A is 2×3 and order of matrix B is 3×5 then order of matrix $B'A'$ is : 1

(a)5×2 (b) 2×5 (c) 5×3 (d) 3×2

(vi) If $f(x) = \begin{cases} kx + 1, & x \le 5 \\ 3x - 5, & x > 5 \end{cases}$ is continuous then value of k is : 1

(a)$\frac{9}{5}$ (b) $\frac{5}{9}$ (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

(vii) $\frac{d}{dx}\{\tan^{-1}(e^x)\}$ is equal to : 1

(a)$e^x \tan^{-1} e^x$ (b) $\frac{e^x}{1+e^{2x}}$ (c) 0 (d)$e^x \sec^{-1} x$

(viii) Slope of tangent to the curve $y = x^2 - 2x + 1$ at $x = 3$ is: 1

(a)4 (b)6 (c)0 (d)2

(ix) $\int 3x^2 dx$ is equal to : 1

(a)$x + c$ (b)$x^2 + c$ (c)$x^3 + c$ (d)$x^4 + c$

(x) $\int_0^{\pi/2} \frac{\sin^{1/2} x}{\sin^{1/2} x + \cos^{1/2} x} dx$ is equal to : 1

(a)0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

(xi) Degree of differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = 0$ is : 1

(a)3 (b) 2 (c)1 (d) 0

(xii) If $\vec{a}.\vec{b} = |\vec{a} \times \vec{b}|$ then angle between vector $\vec{a}$ and vector $\vec{b}$ is : 1

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

(xiii) If $\vec{a}.\vec{b} = 0$ then angle between vectors $\vec{a}$ and $\vec{b}$ is : 1

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

(xiv) Direction ratios of line given by $\frac{x-1}{3} = \frac{2y+6}{12} = \frac{1-z}{-7}$ are : 1

(a) $< 3,12,-7 >$ (b) $< 3,-6,7 >$ (c) $< 3,6,7 >$ (d)$< 3,6,-7 >$

(xv) Maximum value of $Z = 3x + y$ for the constraints $x + y \le 4$, $x \ge 0$, $y \ge 0$ is: 1

(a)12 (b)16 (c)4 (d)10

(xvi) If $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{8}$ and $P(A \cap B) = \frac{1}{5}$ then $P(A|B)$ is equal to : 1

(a)$\frac{2}{5}$ (b) $\frac{8}{15}$ (c) $\frac{2}{3}$ (d) $\frac{5}{8}$

Q2 Fill in the blanks from the given options

0, 1, $< 3, -1, 2 >$, $\frac{\pi}{2}$, 6, 2, 5, 4, $-\sin x$, $\tan x$

(i) Value of $\sin^{-1}(1)$ is ____________ 1

(ii) If $A = [a_{ij}]_{2\times 3}$ such that $a_{ij} = i + j$ then $a_{11} =$___________ 1

(iii) If $\begin{vmatrix} x & 0 \\ 7 & 1 \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 7 & 2 \end{vmatrix}$ then $x =$ _______ 1

(iv) If $y = \cos x$ then at $x = 0$, $\frac{dy}{dx} =$ ______________ 1

(v) $\int_0^5 dx =$___________ 1

(vi) Order of the differential equation $\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^3 + y = 0$ is __________ 1

(vii) Direction ratios of a line which is perpendicular to the plane $3x - y + 2z = 9$ are _______________ 1

(viii) Probability of occurrence of impossible event = _______________ 1

Q3 State true or false for the following statements :

(i) If A is a square matrix then $(A + A')$ is a skew-symmetric matrix. 1

(ii) If $y = 10x$ then $\frac{dy}{dx} = 0$. 1

(iii) If $y = \tan x$ then $\frac{dy}{dx} = \sec^2 x$ 1

(iv) $\int dx = x^2 + c$ 1

(v) $xdy - ydx = 0$ is a variable separable type of differential equation. 1

(vi) Scalar product of two perpendicular vectors is zero. 1

(vii) Point $(3, -4, 2)$ lies in the plane $2x + y - z = 0$ 1

(viii) If $P(E) = 0.4$ then $P(not\ E) = 0.6$ 1

Section – B

Q4 If $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ and $f(x) = x^2 + 2x + 3$ then find $f(A)$. 2

Q5 Find the interval in which function $f(x) = x^2 + 2x - 7$ is increasing. 2

OR

Find the slope of the normal to the curve $y = x^3 - x + 1$ at the point whose x –coordinate is 2 . 2

Q6 Evaluate $\int e^x \left(\log x + \frac{1}{x}\right) dx$. 2

OR

Evaluate $\int x \sin x \, dx$ 2

Q7 Using integration find the area bounded by the parabola $y^2 = 4x$ straight lines $x = 1, x = 4$ in the first quadrant. 2

Q8 Find the unit vector in the direction of diagonal of the parallelogram whose sides are given by the vectors $\vec{a} = 2\hat{\imath} - \hat{\jmath} - 3\hat{k}$, $\vec{b} = 5\hat{\imath} + 2\hat{\jmath} - \hat{k}$ 2

OR

If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$, $\vec{b} = 7\hat{\imath} - 2\hat{\jmath} - 4\hat{k}$ then find $\vec{a} \times \vec{b}$. 2

Section – C

Q9 Find the value of: $2\tan^{-1}(1) - \cos^{-1}\left(\frac{-1}{2}\right) + 3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ 4

Q10 If $y = x^{\sin x} + (\sin x)^x$ then find $\frac{dy}{dx}$. **4**

OR

If $y = (\tan^{-1} x)^2$, show that $(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$ **4**

Q11 Evaluate $\int \frac{dx}{(x-1)(x-2)(x-3)}$. **4**

OR

Evaluate $\int \frac{\sec^2 x}{\tan^2 x - 4\tan x + 7} dx$ **4**

Q12 Find the general solution of the differential equation $x^2 dy - (x^2 + xy + y^2)dx = 0$. **4**

OR

Find the general solution of the differential equation $\sec^2 x \tan y\, dx - \sec^2 y \tan x\, dy = 0$. **4**

Q13 Bag I contains 3 red and 4 white balls. Bag II contains 7 red and 5 white balls. A bag is selected at random and a ball is drawn from it, which is found to be red. Find the probability that ball is drawn from bag II.

Section – D

Q14 Solve the following system of linear equations by matrix method : **6**

$2x + 3y - 5z = 13$, $x - y + z = -2$, $3x + 2y - z = 8$

OR

Express $A = \begin{bmatrix} 2 & 3 & 5 \\ 0 & 2 & 9 \\ 3 & 2 & 8 \end{bmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix. **6**

Q15 Find the shortest distance between the lines **6**

$\vec{r} = 6\hat{i} - \hat{j} + 3\hat{k} + \lambda(\hat{i} + 3\hat{j} + 2\hat{k})$ and $\vec{r} = 9\hat{i} + \hat{j} - 4\hat{k} + \mu(\hat{i} - 2\hat{j} + \hat{k})$

OR

Find the foot of perpendicular drawn from the point $(2, -3, 5)$ on the plane $3x + 4y - 2z = 20$ **6**

Q16 Solve the following linear programming problem graphically:

Maximize and minimize $Z = 4x + 3y$ subject to the constraints **6**

$$x + y \le 8, \quad 4x + y \ge 8, \quad x - y \ge 0, \quad x \ge 0 \quad , y \ge 0$$

OR

Solve the following linear programming problem graphically:

Maximize and minimize $Z = 5x + 2y - 2$ subject to the constraints **6**

$$x + y \le 10, \quad x + y \ge 3, \quad x \le 8, \quad y \le 8, \quad x \ge 0, \quad y \ge 0$$