MATHEMATICS (51)

Aims:

1. To acquire knowledge and understanding of the terms, symbols, concepts, principles, processes, proofs, etc. of mathematics.
2. To develop an understanding of mathematical concepts and their application to further studies in mathematics and science.
3. To develop skills to apply mathematical knowledge to solve real life problems.
4. To develop the necessary skills to work with modern technological devices such as calculators and computers in real life situations.
5. To develop drawing skills, skills of reading tables, charts and graphs.
6. To develop an interest in mathematics.

CLASS IX

*There will be **one** paper of **two and a half** hours duration carrying 80 marks and Internal Assessment of 20 marks.*

Certain questions may require the use of Mathematical tables (Logarithmic and Trigonometric tables).

The solution of a question may require the knowledge of more than one branch of the syllabus.

1. Pure Arithmetic

Rational and Irrational Numbers

Rational, irrational numbers as real numbers, their place in the number system. Surds and rationalization of surds. Simplifying an expression by rationalizing the denominator. Representation of rational and irrational numbers on the number line.

Proofs of irrationality of $\sqrt{2}, \sqrt{3}\ \sqrt{5}$

2. Commercial Mathematics

Compound Interest

(a) Compound interest as a repeated Simple Interest computation with a growing Principal. Use of this in computing Amount over a period of 2 or 3 years.

(b) Use of formula $A = P(1 + \frac{r}{100})^n$. Finding CI from the relation CI = A – P.

- *Interest compounded half-yearly included.*
- *Using the formula to find one quantity given different combinations of A, P, r, n, CI and SI; difference between CI and SI type included. Rate of growth and depreciation.*

Note: Paying back in equal installments, being given rate of interest and installment amount, **not included**.

3. Algebra

(i) Expansions

Recall of concepts learned in earlier classes.

$(a \pm b)^2$

$(a \pm b)^3$

$(x \pm a)\ (x \pm b)$

$(a \pm b \pm c)^2$

(ii) Factorisation

$a^2 - b^2$

$a^3 \pm b^3$

$ax^2 + bx + c$, by splitting the middle term.

(iii) Simultaneous Linear Equations in two variables. (With numerical coefficients only)

- *Solving algebraically by:*
  - *Elimination*
  - *Substitution and*
  - *Cross Multiplication method*
- *Solving simple problems by framing appropriate equations.*

(iv) Indices/ Exponents

Handling positive, fractional, negative and "zero" indices.

Simplification of expressions involving various exponents

$a^m \times a^n = a^{m+n}, a^m \div a^n = a^{m-n}, (a^m)^n = a^{mn}$ *etc. Use of laws of exponents.*

(v) Logarithms

(a) Logarithmic form vis-à-vis exponential form: interchanging.

(b) Laws of Logarithms and their uses.

Expansion of expression with the help of laws of logarithms

e.g. $y = \frac{a^4 \times b^2}{c^3}$

log y = 4 log a + 2 log b – 3 log c etc.

4. Geometry

(i) Triangles

(a) Congruency: four cases: SSS, SAS, AAS, and RHS. Illustration through cutouts. Simple applications.

(b) Problems based on:

- *Angles opposite equal sides are equal and converse.*
- *If two sides of a triangle are unequal, then the greater angle is opposite the greater side and converse.*
- *Sum of any two sides of a triangle is greater than the third side.*
- *Of all straight lines that can be drawn to a given line from a point outside it, the perpendicular is the shortest.*

Proofs not required.

(c) Mid-Point Theorem and its converse, equal intercept theorem

(i) Proof and simple applications of mid-point theorem and its converse.

(ii) Equal intercept theorem: proof and simple application.

(d) Pythagoras Theorem

Area based proof and simple applications of Pythagoras Theorem and its converse.

(ii) Rectilinear Figures

(a) Proof and use of theorems on parallelogram.

- *Both pairs of opposite sides equal (without proof).*
- *Both pairs of opposite angles equal.*
- *One pair of opposite sides equal and parallel (without proof).*
- *Diagonals bisect each other and bisect the parallelogram.*
- *Rhombus as a special parallelogram whose diagonals meet at right angles.*
- *In a rectangle, diagonals are equal, in a square they are equal and meet at right angles.*

(b) Constructions of Polygons

Construction of quadrilaterals (including parallelograms and rhombus) and regular hexagon using ruler and compasses only.

(c) Proof and use of Area theorems on parallelograms:

- *Parallelograms on the same base and between the same parallels are equal in area.*
- *The area of a triangle is half that of a parallelogram on the same base and between the same parallels.*
- *Triangles between the same base and between the same parallels are equal in area (without proof).*
- *Triangles with equal areas on the same bases have equal corresponding altitudes.*

(iii) Circle:

(a) Chord properties

- *A straight line drawn from the centre of a circle to bisect a chord which is not a diameter is at right angles to the chord.*

- *The perpendicular to a chord from the centre bisects the chord (without proof).*
- *Equal chords are equidistant from the centre.*
- *Chords equidistant from the centre are equal (without proof).*
- *There is one and only one circle that passes through three given points not in a straight line.*

(b) Arc and chord properties:

- *If two arcs subtend equal angles at the centre, they are equal, and its converse.*
- *If two chords are equal, they cut off equal arcs, and its converse (without proof).*

Note: Proofs of the theorems given above are to be taught unless specified otherwise.

5. Statistics

Introduction, collection of data, presentation of data, Graphical representation of data, Mean, Median of ungrouped data.

(i) Understanding and recognition of raw, arrayed and grouped data.

(ii) Tabulation of raw data using tally-marks.

(iii) Understanding and recognition of discrete and continuous variables.

(iv) Mean, median of ungrouped data.

(v) Class intervals, class boundaries and limits, frequency, frequency table, class size for grouped data.

(vi) Grouped frequency distributions: the need to and how to convert discontinuous intervals to continuous intervals.

(vii) Drawing a frequency polygon.

6. Mensuration

Area and perimeter of a triangle and a quadrilateral. Area and circumference of circle. Surface area and volume of Cube and Cuboids.

(a) Area and perimeter of triangle (including Heron's formula), all types of Quadrilaterals.

(b) Circle: Area and Circumference. Direct application problems including Inner and Outer area.

Areas of sectors of circles other than quarter-circle and semicircle are not included.

(c) Surface area and volume of 3-D solids: cube and cuboid including problems of type involving:

- *Different internal and external dimensions of the solid.*
- *Cost.*
- *Concept of volume being equal to area of cross-section* x *height.*
- *Open/closed cubes/cuboids.*

7. Trigonometry

(a) Trigonometric Ratios: sine, cosine, tangent of an angle and their reciprocals.

(b) Trigonometric ratios of standard angles - 0, 30, 45, 60, 90 degrees. Evaluation of an expression involving these ratios.

(c) Simple 2-D problems involving one right-angled triangle.

(d) Concept of trigonometric ratios of complementary angles and their direct application:

$\sin A = \cos (90 - A)$, $\cos A = \sin (90 – A)$

$\tan A = \cot (90 – A)$, $\cot A = \tan (90- A)$

$\sec A = \operatorname{cosec} (90 – A)$, $\operatorname{cosec} A=\sec (90 – A)$

8. Coordinate Geometry

Cartesian System, plotting of points in the plane for given coordinates, solving simultaneous linear equations in 2 variables graphically and finding the distance between two points using distance formula.

(a) Dependent and independent variables.

(b) Ordered pairs, coordinates of points and plotting them in the Cartesian plane.

(c) *Solution of Simultaneous Linear Equations graphically.*

(d) *Distance formula.*

INTERNAL ASSESSMENT

A minimum of two assignments are to be done during the year as prescribed by the teacher.

Suggested Assignments

- Conduct a survey of a group of students and represent it graphically - height, weight, number of family members, pocket money, etc.
- Planning delivery routes for a postman/milkman.
- Running a tuck shop/canteen.
- Study ways of raising a loan to buy a car or house, e.g. bank loan or purchase a refrigerator or a television set through hire purchase.
- Cutting a circle into equal sections of a small central angle to find the area of a circle by using the formula $A = \pi r^2$.
- To use flat cutouts to form cube, cuboids and pyramids to obtain formulae for volume and total surface area.
- Draw a circle of radius r on a ½ cm graph paper, and then on a 2 mm graph paper. Estimate the area enclosed in each case by actually counting the squares. Now try out with circles of different radii. Establish the pattern, if any, between the two observed values and the theoretical value (area = πr^2). Any modifications?

CLASS X

*There will be **one** paper of **two and a half** hours duration carrying 80 marks and Internal Assessment of 20 marks.*

Certain questions may require the use of Mathematical tables (Logarithmic and Trigonometric tables).

1. Commercial Mathematics

(i) Goods and Services Tax (GST)

Computation of tax including problems involving discounts, list-price, profit, loss, basic/cost price including inverse cases. Candidates are also expected to find price paid by the consumer after paying State Goods and Service Tax (SGST) and Central Goods and Service Tax (CGST) - the different rates as in vogue on different types of items will be provided. Problems based on corresponding inverse cases are also included.

(ii) Banking

Recurring Deposit Accounts*: computation of interest and maturity value using the formula:*

$$I = P\frac{n(n+1)}{2\times 12}\times\frac{r}{100}$$

$$MV = P\,x\,n + I$$

(iii) Shares and Dividends

(a) Face/Nominal Value, Market Value, Dividend, Rate of Dividend, Premium.

(b) Formulae

- *Income = number of shares × rate of dividend × FV.*
- *Return = (Income / Investment) × 100.*

Note:** Brokerage and fractional shares **not included.

2. Algebra

(i) Linear Inequations

Linear Inequations in one unknown for x ∈ N, W, Z, R. Solving:

- *Algebraically and writing the solution in set notation form.*
- *Representation of solution on the number line.*

(ii) Quadratic Equations in one variable

(a) Nature of roots

- *Two distinct real roots if* $b^2 - 4ac > 0$
- *Two equal real roots if* $b^2 - 4ac = 0$
- *No real roots if* $b^2 - 4ac < 0$

(b) Solving Quadratic equations by:

- *Factorisation*
- *Using Formula.*

(c) Solving simple quadratic equation problems.

(iii) Ratio and Proportion

(a) Proportion, Continued proportion, mean proportion

(b) Componendo, dividendo, alternendo, invertendo properties and their combinations.

(c) Direct simple applications on proportions only.

(iv) Factorisation of polynomials:

(a) Factor Theorem.

(b) Remainder Theorem.

(c) Factorising a polynomial completely after obtaining one factor by factor theorem.

Note: f (x) not to exceed degree 3.

(v) Matrices

(a) Order of a matrix. Row and column matrices.

(b) Compatibility for addition and multiplication.

(c) Null and Identity matrices.

(d) Addition and subtraction of 2×2 matrices.

(e) *Multiplication of a 2×2 matrix by*

- *a non-zero rational number*
- *a matrix.*

(vi) Arithmetic and Geometric Progression

- *Finding their General term.*
- *Finding Sum of their first 'n' terms.*
- *Simple Applications.*

(vii) Co-ordinate Geometry

(a) Reflection

(i) Reflection of a point in a line:

x=0, y =0, x= a, y=a, the origin.

(ii) Reflection of a point in the origin.

(iii) Invariant points.

(b) Co-ordinates expressed as (*x,y*), Section formula, Midpoint formula, Concept of slope, equation of a line, Various forms of straight lines.

(i) Section and Mid-point formula (Internal section only, co-ordinates of the centroid of a triangle included).

(ii) Equation of a line:

- *Slope –intercept form* $y = mx + c$
- *Two- point form* $(y-y_1) = m(x-x_1)$
- *Geometric understanding of 'm' as slope/ gradient/* $\tan\theta$ *where* θ *is the angle the line makes with the positive direction of the x axis.*
- *Geometric understanding of 'c' as the y-intercept/the ordinate of the point where the line intercepts the y axis/ the point on the line where x=0.*
- *Conditions for two lines to be parallel or perpendicular.*

Simple applications of all the above.

3. Geometry

(a) Similarity

Similarity, conditions of similar triangles.

(i) As a size transformation.

(ii) Comparison with congruency, keyword being proportionality.

(iii) Three conditions: SSS, SAS, AA. Simple applications (proof not included).

(iv) Applications of Basic Proportionality Theorem.

(v) Areas of similar triangles are proportional to the squares of corresponding sides.

(vi) Direct applications based on the above including applications to maps and models.

(b) Loci

Loci: Definition, meaning, Theorems and constructions based on Loci.

(i) *The locus of a point at a fixed distance from a fixed point is a circle with the fixed point as centre and fixed distance as radius.*

(ii) The locus of a point equidistant from two intersecting lines is the bisector of the angles between the lines.

(iii) The locus of a point equidistant from two given points is the perpendicular bisector of the line joining the points.

Proofs not required.

(c) Circles

(i) Angle Properties

- *The angle that an arc of a circle subtends at the centre is double that which it subtends at any point on the remaining part of the circle.*
- *Angles in the same segment of a circle are equal (without proof).*
- *Angle in a semi-circle is a right angle.*

(ii) Cyclic Properties:

- *Opposite angles of a cyclic quadrilateral are supplementary.*
- *The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle (without proof).*

(iii) Tangent and Secant Properties:

- *The tangent at any point of a circle and the radius through the point are perpendicular to each other.*
- *If two circles touch, the point of contact lies on the straight line joining their centres.*
- *From any point outside a circle, two tangents can be drawn, and they are equal in length.*
- *If two chords intersect internally or externally then the product of the lengths of the segments are equal.*
- *If a chord and a tangent intersect externally, then the product of the lengths of segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.*
- *If a line touches a circle and from the point of contact, a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.*

Note: Proofs of the theorems given above are to be taught unless specified otherwise.

(iv) Constructions

(a) Construction of tangents to a circle from an external point.

(b) Circumscribing and inscribing a circle on a triangle and a regular hexagon.

4. Mensuration

Area and volume of solids – Cylinder, Cone and Sphere.

Three-dimensional solids - right circular cylinder, right circular cone and sphere: Area (total surface and curved surface) and Volume. Direct application problems including cost, Inner and Outer volume and melting and recasting method to find the volume or surface area of a new solid. Combination of solids included.

Note: Problems on Frustum are not included.

5. Trigonometry

(a) Using Identities to solve/prove simple algebraic trigonometric expressions

$\sin^2 A + \cos^2 A = 1$

$1 + \tan^2 A = \sec^2 A$

$1 + \cot^2 A = \operatorname{cosec}^2 A;\ 0 \le A \le 90°$

(b) *Heights and distances: Solving 2-D problems involving angles of elevation and depression using trigonometric tables.*

Note: Cases involving more than two right angled triangles excluded.

6. Statistics

Statistics – basic concepts, Mean, Median, Mode. Histograms and Ogive.

(a) Computation of:

- *Measures of Central Tendency: Mean, median, mode for raw and arrayed data. Mean*, median class and modal class for grouped data. (both continuous and discontinuous).*

** Mean by all 3 methods included:*

Direct : $\frac{\Sigma fx}{\Sigma f}$

Short-cut : $A + \frac{\Sigma fd}{\Sigma f}$ where $d = x - A$

Step-deviation: $A + \frac{\Sigma ft}{\Sigma f} \times i$ where $t = \frac{x - A}{i}$

(b) Graphical Representation. Histograms and Less than Ogive.

- *Finding the mode from the histogram, the upper quartile, lower Quartile and median etc. from the ogive.*
- *Calculation of inter Quartile range.*

7. Probability

Random experiments, Sample space, Events, definition of probability, Simple problems on single events.

SI UNITS, SIGNS, SYMBOLS AND ABBREVIATIONS

(1) Agreed conventions

(a) Units may be written in full or using the agreed symbols, but no other abbreviation may be used.

(b) The letter 's' is never added to symbols to indicate the plural form.

(c) A full stop is not written after symbols for units unless it occurs at the end of a sentence.

(d) When unit symbols are combined as a quotient, *e.g.,* metre per second, it is recommended that it should be written as m/s, or as m s^{-1}.

(e) Three decimal signs are in common international use: the full point, the mid-point and the comma. Since the full point is sometimes used for multiplication and the comma for spacing digits in large numbers, it is recommended that the mid-point be used for decimals.

(2) Names and symbols

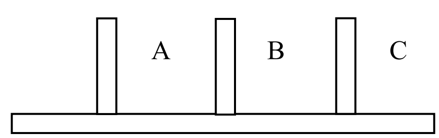
In general			
Implies that	⇒	is logically equivalent to	⇔
Identically equal to	≡	is approximately equal to	>>
In set language			
Belongs to	∈	does not belong to	∉
is equivalent to	↔	is not equivalent to	↮
union	∪	intersection	∩
universal set	ξ	is contained in	⊂
natural (counting) numbers	N	the empty set	ø
		whole numbers	W
integers	Z	real numbers	R
In measures			
Kilometre	km	Metre	m
Centimetre	cm	Millimetre	mm
Kilogram	kg	Gram	g
Litre	L	Centilitre	cL
square kilometre	km^2	Square meter	m^2
square centimetre	cm^2	Hectare	ha
cubic metre	m^3	Cubic centimetre	cm^3
kilometres per hour	km/h	Metres per second	m/s

INTERNAL ASSESSMENT

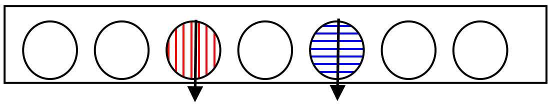
The minimum number of assignments: Two assignments as prescribed by the teacher.

Suggested Assignments

- Comparative newspaper coverage of different items.
- Survey of various types of Bank accounts, rates of interest offered.
- Planning a home budget.
- Conduct a survey in your locality to study the mode of conveyance / Price of various essential commodities / favourite sports. Represent the data using a bar graph / histogram and estimate the mode.
- To use a newspaper to study and report on shares and dividends.
- Set up a dropper with ink in it vertical at a height say 20 cm above a horizontally placed sheet of plain paper. Release one ink drop; observe the pattern, if any, on the paper. Vary the vertical distance and repeat. Discover any pattern of relationship between the vertical height and the ink drop observed.
- You are provided (or you construct a model as shown) - three vertical sticks (size of a pencil) stuck to a horizontal board. You should also have discs of varying sizes with holes (like a doughnut). Start with one disc; place it on (in) stick A. Transfer it to another stick (B or C); this is one move (m). Now try with two discs placed in A such that the large disc is below, and the smaller disc is above (number of discs = n=2 now). Now transfer them one at a time in B or C to obtain similar situation (larger disc below). How many moves? Try with more discs (n = 1, 2, 3, etc.) and generalise.



- The board has some holes to hold marbles, red on one side and blue on the other. Start with one pair. Interchange the positions by making one move at a time. A marble can jump over another to fill the hole behind. The move (m) equal 3. Try with 2 (n=2) and more. Find the relationship between n and m.



- Take a square sheet of paper of side 10 cm. Four small squares are to be cut from the corners of the square sheet and then the paper folded at the cuts to form an open box. What should be the size of the squares cut so that the volume of the open box is maximum?

- Take an open box, four sets of marbles (ensuring that marbles in each set are of the same size) and some water. By placing the marbles and water in the box, attempt to answer the question: do larger marbles or smaller marbles occupy more volume in a given space?

- An eccentric artist says that the best paintings have the same area as their perimeter (numerically). Let us not argue whether such sizes increase the viewer's appreciation, but only try and find what sides (in integers only) a rectangle must have if its area and perimeter are to be equal (Note: there are only two such rectangles).

- Find by construction the centre of a circle, using only a 60-30 setsquare and a pencil.

- Various types of "cryptarithm".

EVALUATION

The assignments/project work are to be evaluated by the subject teacher and by an External Examiner. (The External Examiner may be a teacher nominated by the Head of the school, who could be from the faculty, **but not teaching the subject in the section/class**. For example, a teacher of Mathematics of Class VIII may be deputed to be an External Examiner for Class X, Mathematics projects.)

The Internal Examiner and the External Examiner will assess the assignments independently.

Award of Marks	**(20 Marks)**
Subject Teacher (Internal Examiner)	10 marks
External Examiner	10 marks

The total marks obtained out of 20 are to be sent to CISCE by the Head of the school.

The Head of the school will be responsible for the online entry of marks on CISCE's CAREERS portal by the due date.

INTERNAL ASSESSMENT IN MATHEMATICS - GUIDELINES FOR MARKING WITH GRADES

Criteria	Preparation	Concepts	Computation	Presentation	Understanding	Marks
Grade I	Exhibits and selects a well-defined problem. Appropriate use of techniques.	Admirable use of mathematical concepts and methods and exhibits competency in using extensive range of mathematical techniques.	Careful and accurate work with appropriate computation, construction and measurement with correct units.	Presents well stated conclusions; uses effective mathematical language, symbols, conventions, tables, diagrams, graphs, etc.	Shows strong personal contribution; demonstrate knowledge and understanding of assignment and can apply the same in different situations.	4 marks for each criterion
Grade II	Exhibits and selects routine approach. Fairly good techniques.	Appropriate use of mathematical concepts and methods and shows adequate competency in using limited range of techniques.	Commits negligible errors in computation, construction and measurement.	Some statements of conclusions; uses appropriate math language, symbols, conventions, tables, diagrams, graphs, etc.	Neat with average amount of help; assignment shows learning of mathematics with a limited ability to use it.	3 marks for each criterion
Grade III	Exhibits and selects trivial problems. Satisfactory techniques.	Uses appropriate mathematical concepts and shows competency in using limited range of techniques.	Commits a few errors in computation, construction and measurement.	Assignment is presentable though it is disorganized in some places.	Lack of ability to conclude without help; shows some learning of mathematics with a limited ability to use it.	2 marks for each criterion
Grade IV	Exhibits and selects an insignificant problem. Uses some unsuitable techniques.	Uses inappropriate mathematical concepts for the assignment.	Commits many mistakes in computation, construction and measurement.	Presentation made is somewhat disorganized and untidy.	Lack of ability to conclude even with considerable help; assignment contributes to mathematical learning to a certain extent.	1 mark for each criterion
Grade V	Exhibits and selects a completely irrelevant problem. Uses unsuitable techniques.	Not able to use mathematical concepts.	Inaccurate computation, construction and measurement.	Presentation made is completely disorganized, untidy and poor.	Assignment does not contribute to mathematical learning and lacks practical applicability.	0 mark